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How estuaries work: A Charlotte Harbor example

by Robert H. Weisberg$^1$ and Lianyuan Zheng$^1$

ABSTRACT

We consider the time-averaged, estuarine circulation through a control volume analysis of its mechanical energy balance; hence the title of our paper: “How estuaries work.” The venue is the Charlotte Harbor on Florida’s west coast, and the analysis medium is the primitive equation ECOM3D-si model. We are motivated by an estuary classification dilemma. Being that estuarine circulation is buoyancy driven by the horizontal salinity gradient, and that the gradient increases with increasing vertical mixing as the salinity isolines rotate from horizontal to vertical, it follows that an increase in mixing should lead to an increase in circulation. Yet, at some point, intermediate between partial and well mixed, the estuarine circulation decreases. We explore this conundrum by considering the sources and sinks of mechanical energy, derived from river, tide, and wind forcing, through analyses of pressure work, work by stresses operating on the surface and the bottom, work against buoyancy through mixing and advection, and turbulence energy production by velocity component shears. How the energy partitions between work against buoyancy (that sets the driving force) and turbulence production (that serves as a brake) provides an answer. Depending on tides and winds, if the incremental rate of turbulence production exceeds that of buoyancy work, the estuarine circulation slows. A useful measure is the ratio of buoyancy work to turbulence production. As this ratio increases so does the circulation, and conversely. For the Charlotte Harbor, tides alone cause peak circulation. Weak, oscillatory winds may add to this, but with increasing wind stress the tide and wind-averaged estuarine circulation decreases. Further exploration of these effects across Rayleigh number space with varying estuarine geometries and forcing functions may lead to improved understandings of estuary workings in general.

1. Introduction

Estuaries provide the transitions from rivers to oceans. Fluid parcels originating with zero salinity at the river mouth exit the estuary with salinity approaching that of the coastal ocean, and the resulting density changes cause pressure gradients that drive a nontidal circulation, generally referred to as “estuarine circulation.” Seminal works on estuarine circulation for the Chesapeake Bay and its tributaries are summarized in Cameron and Pritchard (1963). They argue for the circulation as follows. In the absence of tidal mixing, freshwater flows seaward on top of a salt-water wedge. Turbulent entrainment causes a net transport of seawater into the fresh water lens that increases the upper layer transport in excess of the river discharge rate. A net landward transport must then ensue in the salt

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wedge to conserve both water and salt. The addition of tides increases the turbulent entrainment, the resulting horizontal pressure gradient, and hence the estuarine circulation, and the estuary transforms from a salt wedge to partially mixed. This line of reasoning, of increasing mixing leading to increasing circulation, eventually transforms a partially mixed estuary to a well mixed estuary as the salinity (density) isolines rotate from being nearly horizontal for the salt wedge to nearly vertical for the well mixed cases. Pritchard (1954, 1956) discusses the salinity and horizontal momentum balances accompanying these heuristic arguments.

A degree of formalism was afforded this classification by Hansen and Rattray (1966) via a similarity solution to a set of equations governing estuarine circulation (Hansen and Rattray, 1965). A classification diagram was constructed based on two nondimensional parameters, a circulation parameter defined as the ratio of the mean surface current to the river discharge divided by the cross-sectional area, and a stratification parameter defined as the vertical salinity difference divided by the cross-section averaged salinity, and various estuaries were classified. Their solution also included wind effects, but lack of observations precluded testing.

The foregoing concepts are readily observed in an exact solution to the Navier-Stokes equations for the special case of depth-independent horizontal salinity gradient and constant vertical mixing coefficient (e.g., Officer, 1976). Consider the force balances between the horizontal pressure gradient and friction in the estuary’s along-axis direction and between the vertical pressure gradient and gravity in the vertical direction. These are:

\[
\frac{1}{\rho} \frac{\partial p}{\partial y} = K \frac{\partial^2 v}{\partial z^2} \tag{1}
\]

\[
\frac{1}{\rho} \frac{\partial p}{\partial z} = -g \tag{2}
\]

where \(p\) is pressure, \(\rho\) is density, \(y(v)\) is the along-axis direction (velocity component), \(z\) is the vertical direction, \(g\) is the gravitational acceleration, and \(K\) is a constant vertical eddy diffusivity for momentum. If the along-axis density (salinity) gradient is assumed to be independent of depth, then these equations may be combined and vertically integrated using the boundary conditions:

\[
v = 0 \quad \text{at} \quad z = 0 \quad (\text{no slip at the bottom}) \tag{3}
\]

and

\[
\frac{\partial v}{\partial z} = 0 \quad \text{at} \quad z = \zeta \quad (\text{no stress at the sea surface}). \tag{4}
\]

Constraining the volume flux to equal the river discharge rate \((R)\) yields a solution for the vertical distribution of the estuarine circulation in terms of the horizontal density gradient:

\[
v(z) = \frac{3R}{\zeta W} \left[ \frac{z}{\zeta} - \frac{1}{2} \left( \frac{z}{\zeta} \right)^2 \right] + \frac{1}{48} \frac{g}{\rho K} \cdot \frac{\partial p}{\partial y} \cdot \zeta^3 \left[ - \frac{6z}{\zeta} + 15 \cdot \left( \frac{z}{\zeta} \right)^2 - 8 \cdot \left( \frac{z}{\zeta} \right)^3 \right] \tag{5}
\]
where \( W \) is the width of the estuary. Evaluating this expression at \( z = \zeta \) results in the along-axis surface flow:

\[
n(\zeta) = \frac{3R}{2\zeta W} + \frac{1}{48} \frac{g}{\rho K} \cdot \frac{\partial \rho}{\partial y} \cdot \zeta^3
\]

(6)

where the first term is the barotropic effect of the river inflow and the second term is the baroclinic effect of the along-axis density gradient.

These equations may also be solved for the along-axis surface slope, \((\partial \zeta / \partial y)\), which follows from the volume flux constraint:

\[
\frac{\partial \zeta}{\partial y} = -\frac{3K}{g\zeta^3} \cdot \frac{R}{W} - \frac{3\zeta}{8\rho} \cdot \frac{\partial \rho}{\partial y}
\]

(7)

and the distribution of the pressure gradient force with depth is then:

\[
\frac{\partial \rho}{\partial y} = -\frac{3\rho KR}{W\zeta^3} + g\zeta \cdot \frac{\partial \rho}{\partial y} \cdot \left( \frac{5}{8} - \frac{z}{\zeta} \right)
\]

(8)

showing that the contribution to the pressure gradient force by the along-axis density gradient changes sign at a distance of \( z = (5/8)\zeta \) off the bottom. This reversal accounts for the outflow (inflow) of fluid in the upper (lower) layer of the estuary.

The foregoing solution implies increasing estuarine circulation with increasing horizontal salinity gradient. Yet, Cameron and Pritchard (1963) argue for a decrease in circulation and a small unidirectional flow over the entire water column under well mixed conditions when the horizontal salinity gradient is a maximum. How this may be reconciled, or in other words, how estuaries work, is the topic of our paper.

Eq. (5) contains a partial answer. While the estuarine circulation is directly proportional to the along-axis salinity gradient, it is inversely proportional to the eddy diffusivity. If the eddy coefficient increases at a rate more rapid than the salinity gradient, then the circulation decreases. This is also implied by several laboratory and numerical studies. Linden and Simpson (1986) examined the effects of turbulence on gravity currents in laboratory, lock exchange experiments. The exchange flow decreases with increasing turbulence in response to the vertical transport of horizontal momentum and the decrease in the horizontal density gradient. Numerical experiments of lock exchange flow past a constriction by Hogg et al. (2001) show a similar reduction in flow with increasing turbulent mixing. MacCready (1999) used an idealized, two layered, analytical model to study the effects of rivers and tidal mixing on estuarine circulation. Increasing mixing causes a decrease in estuarine exchange flow (defined as one half times the upper layer minus the lower layer mean flows) on a time scale associated with the vertical diffusion of momentum. Accompanying the reduced exchange flow, however, is a decrease in the up-estuary salt flux and consequently an increase in the along-axis salinity gradient. This negative feedback then tends to increase the exchange flow back toward its original value as advanced by Park and Kuo (1996) in a numerical model study of the Rappahannock.
River estuary. A generalized numerical model study by Li (1999) also shows a decrease in estuarine circulation in response to an increase in vertical mixing.

On the basis of the Hansen and Rattray (1966) solution (or alternatively from Eq. 5), Park and Kuo (1996) introduced a nondimensional parameter, $\gamma$, as the fractional change in the ratio between the along-axis salinity gradient and the vertical eddy diffusivity under cases of neap and spring tides (corresponding to decreased or increased vertical mixing, respectively). They argue that as vertical mixing increases the estuarine circulation will either increase, or decrease depending on whether $\gamma > 1$, or $< 1$, respectively, i.e., whether the increase in density gradient is greater than or less than the increase in eddy diffusivity.

Our findings are similar to those given above, but it must be recognized that estuarine circulation is considerably more complex. Along with rivers and tides, winds also contribute to the circulation and mixing. Weisberg and Sturges (1976) and Weisberg (1976a) provide evidence for this in multiple and single channel estuary sections, respectively, and, consistent with Hansen and Rattray (1965), these wind effects may be much larger in magnitude than that of gravitational convection. Indirect effects of winds, through adjacent coastal ocean sea level set up, are also important (Wang, 1979).

Mixing also arises through a variety of processes. Along with mixing caused by instability at interfaces and by turbulence production by shears originating from tidal friction along the bottom and wind friction along the surface, contributions also occur through transverse secondary circulations caused by stratified flows over topography (e.g., Geyer, 1993; Chant and Wilson, 1997) and through density field straining caused by stratified flows within boundary layers (e.g., Simpson et al., 1990). Straining adds tidal asymmetry to turbulent mixing through its tendency to increase (decrease) vertical stratification during ebb (flood) tide. Overstraining may further promote mixing by static instability. Stacey et al. (2001) provide evidence for the importance of such straining in estuarine circulation. All of these complicating factors suggest an integrated approach to the problem in order to avoid the intractable details.

2. Experimental design and analyses

We approach the problem through a control volume analysis of the mechanical energy equation. The venue is the Charlotte Harbor (CH) estuary on Florida’s west coast, whose classification may vary from salt wedge to well mixed depending on location and time of year. Periods of peak river discharge and minimal wind stress (summer) may leave the upper reaches of the CH in salt wedge conditions, whereas periods of low river discharge and high wind stress (winter) may result in well-mixed conditions everywhere. The experimental medium is the primitive equation, ECOM3D-si model of Blumberg (1993). Figures 1a and 1b show the study region and the model grid, respectively. The main body of the CH estuary, with the deepest depths and the simplest geometry, is the northern portion fed by the Peace and Myakka rivers. This is our analysis region as designated by the control volume and the two cross-section sampling lines of Figure 1b.

The model is forced by a combination of rivers, tides, and winds. River inflows are from the Peace, Myakka, and Caloosahatchee rivers. We keep these constant at their 1998 spring
season mean flow rates of 165, 40, and 240 m$^3$ s$^{-1}$, respectively. Tidal forcing is from the adjacent Gulf of Mexico. Owing to multiple inlets and a lack of sea level data, the model domain is extended onto the West Florida Shelf (WFS) in order to use a shelf tide model (He and Weisberg, 2002) to force the estuary’s tides. This extended domain also facilitates indirect wind forcing of the estuary via WFS sea level set up. Tidal forcing is by the four constituents ($M_2$, $S_2$, $K_1$, $O_1$) that account for about 95% of the shelf’s tidal variance interpolated onto the Figure 1b open boundary grid. Comparisons of observed and modeled sea level (at Naples, FL, outside the estuary, and at Ft. Myers, FL, inside the estuary) and
velocity hodographs (offshore of Sanibel, FL) in Figures 2a and 2b, respectively, demonstrate the legitimacy of forcing the CH model tides in this way. Wind forcing is imposed uniformly over the model domain. We limit our attention to idealized winds oscillating between southeast and northwest with a periodicity of 8 M2 tidal cycles. This approximately mimics transitions between downwelling- to upwelling-favorable wind conditions with respect to the WFS and between up-estuary to down-estuary winds with respect to the analyzed portion of the CH. Oscillatory, zero mean winds are used so that upon averaging we are left with a net, nontidal and nonwind-driven estuarine circulation that is set up by the combined effects of the river discharge and rectification caused by (river, tide, and wind) mixing.

To determine how rivers, tides, and winds impact the estuarine circulation we analyze experiments forced by rivers only; rivers, plus tides (at values of 0.25, 0.50, 0.75 and 1.00
Figure 2. (a) Comparisons between the computed (solid line) and observed (dashed line) tidal elevation over a period of one month at the Naples and Ft. Myers tide gauge stations. Comparisons are based on summations of the $M_2$, $S_2$, $K_1$, and $O_1$ constituents. (b) Comparisons between the computed (solid line) and observed (dashed line) $M_2$, $S_2$, $K_1$, and $O_1$ tidal constituent velocity hodographs at the EC6 location.
of full amplitude); and rivers, plus full tides, plus winds (at amplitudes of 2.5, 5.0, and 10 m s\(^{-1}\)). The experiments all begin at a state of rest, with river and ocean salinities initialized at 0 and 35, respectively, and with temperature held constant at 20°C. Tidal forcing is imposed (with amplitudes multiplied by 0.0–1.0 for the zero to full tide cases), and the model is spun up for 10 M\(_2\) cycles. Rivers are then turned on, and the model is run for an additional 120 M\(_2\) cycles. Winds are then added, and the model is run for another 80 M\(_2\) cycles (10 complete wind cycles). In all cases, it is this last period of approximately 40 days (over which the solutions are stationary) that is analyzed.

We combine expressions for kinetic and potential energy (Appendix) into a total
mechanical energy \((te)\) equation that is evaluated through independent, term-by-term integrations over the control volume. The governing equation is:

\[
\frac{\partial}{\partial t} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h}^{\xi} t e \, dx \, dy \, dz + \left[ \int_{y_1}^{y_2} \int_{-h}^{\xi} u e \, dy \, dz \right] \bigg|_{x_2}^{x_1} - \left[ \int_{y_1}^{y_2} \int_{-h}^{\xi} u e \, dy \, dz \right] \bigg|_{x_1}
\]

\[\text{(I)}\]

\[+ \left[ \int_{x_1}^{x_2} \int_{-h}^{\xi} v e \, dx \, dz \right] \bigg|_{y_2}^{y_1} - \left[ \int_{x_1}^{x_2} \int_{-h}^{\xi} v e \, dx \, dz \right] \bigg|_{y_1}
\]

\[\text{(II)}\]

\[= - \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h}^{\xi} \left[ \frac{\partial P}{\partial x} u + \frac{\partial P}{\partial y} v \right] \, dx \, dy \, dz
\]

\[\text{(III)}\]

\[+ \int_{x_1}^{x_2} \int_{y_1}^{y_2} \rho_0 (u \tau_{xx} + v \tau_{xy}) \, dx \, dy - \int_{x_1}^{x_2} \int_{y_1}^{y_2} \rho_0 (u \tau_{by} + v \tau_{bx}) \, dx \, dy
\]

\[\text{(IV)}\]

\[- \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h}^{\xi} \rho_0 K_m \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \, dx \, dy \, dz
\]

\[\text{(V)}\]

\[+ \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h}^{\xi} \left( \rho g w - g K_m \frac{\partial P}{\partial z} \right) \, dx \, dy \, dz
\]

\[\text{(VI)}\]

\[- \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h}^{\xi} \rho_0 A_m \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] \, dx \, dy \, dz
\]

\[\text{(VII)}\]

\[+ \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h}^{\xi} A_m \frac{\partial \tau}{\partial x} \, dy \, dz - \int_{y_1}^{y_2} \int_{-h}^{\xi} A_m \frac{\partial \tau}{\partial x} \, dy \, dz \bigg|_{x_2}^{x_1}
\]

\[\text{(VIII)}\]

\[+ \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{-h}^{\xi} A_m \frac{\partial \tau}{\partial y} \, dx \, dz - \int_{x_1}^{x_2} \int_{-h}^{\xi} A_m \frac{\partial \tau}{\partial y} \, dx \, dz \bigg|_{y_2}^{y_1}
\]

\[\text{(IX)}\]
where $x$, $y$, and $z$ ($u$, $v$, and $w$) are the across-axis, along-axis, and vertical coordinates (velocity components), $t$ is time, $\zeta$ is the surface elevation above a mean depth $h(x, y)$, $P$ is pressure, $\rho_0$ and $\rho$ are the reference and perturbation densities, respectively, $g$ is the gravitational acceleration, $\tau$ is stress, the subscript $s$ and $b$ denote the surface and bottom, respectively, and $K_m$ and $A_m$ are the vertical and horizontal eddy viscosity coefficients, respectively. The terms (I–XI) are as follows:
(I) is local change of $te$ integrated over the control volume;
(II) and (III) represent the fluxes of $te$ into the control volume by the horizontal velocity components;
(IV) is the horizontal pressure work;
(V) and (VI) represent work performed by wind stress on the free surface and by friction along the bottom;
(VII) represents energy dissipation by turbulent shear production via the vertical shears of the horizontal velocity components;
(VIII) is the buoyancy flux by vertical advection and diffusion;
(IX) represents energy dissipation by turbulent shear production via the horizontal shears of the horizontal velocity components;
and (X) and (XI) represent the horizontal diffusive fluxes of $te$ into the control volume.

3. Results

We begin by considering the time-averaged (over the last 80 M$^2$ cycles after spin-up to tides and rivers) current and salinity structures in the along-axis and across-axis directions sampled along sections I and II of Figure 1b, respectively. From top to bottom in Figures 3 (section I) and 4 (section II) are panels for the cases of rivers only; rivers, plus 0.5 tides; rivers, plus full tides; and rivers, plus full tides, plus 2.5 m s$^{-1}$, 5.0 m s$^{-1}$, and 10.0 m s$^{-1}$ winds. In the along-axis direction the salinity distribution for the river only case shows salinity isolines packed within the upper 1–2 m atop higher salinity values below and a layered circulation with outflow above inflow. The addition of tides greatly increases the vertical mixing, causing deeper penetration of low salinity and the rotation of salinity isolines from horizontal near the surface to vertical near the bottom. Visually, the magnitude of the circulation for the 0.5 tide case exceeds that for either the river only or the full tide case. Adding oscillatory winds of increasing magnitude further rotates the salinity isolines toward the vertical while decreasing the magnitude of the estuarine circulation. The initial increase in circulation with mixing is consistent with increased buoyancy gradient driving force, whereas the subsequent decrease in circulation with increase in driving force is the conundrum that we are addressing.

The across-axis direction shows similar findings (an increase followed by a decrease in circulation with increasing vertical mixing by tides and winds). In addition we see the effects of the Earth’s rotation with outflowing (near-surface) and inflowing (near-bottom) waters preferring their respective right-hand sides of the estuary. This makes the sampling of specific sections or profiles tenuous with regard to balance closures and hence all the more reason to employ a control volume analysis. These circulation findings may be
Figure 3. Along-axis (section I) nontidal current (left panel) and salinity (right panel) distributions. From top to bottom, the model is forced by rivers only; rivers, plus half tides; rivers, plus full tides; and rivers, plus full tides, plus either 2.5 m s\(^{-1}\), 5.0 m s\(^{-1}\), or 10.0 m s\(^{-1}\) oscillatory (with a period of 8 M\(_2\) tidal cycles) winds. The current and salinity distributions are averaged over 80 M\(_2\) tidal cycles (10 wind cycles), and the contour intervals are 2 cm s\(^{-1}\) and 0.5 for current and salinity, respectively.
summarized through integrations of the sectional mean outflows for all of the cases. While this is section-dependent, we find that the section II transport increases to a peak value at the 0.25 tide case; remains relatively steady through the full tide, plus 2.5 m s$^{-1}$ case; and then decreases for the stronger wind cases (Fig. 5).

We postulate that this behavior depends on vertical mixing. As prelude to the control

Figure 4. As in Figure 3 for the across-axis section II.
volume analyses consider the spatial and temporal dependences of turbulence kinetic energy (TKE) from the ECOM3D-si turbulence closure sub-model (Mellor and Yamada, 1982). Figure 6 shows the time-averaged distributions of TKE for sections I and II under the cases of full tides without winds and full tides with 5.0 m s\(^{-1}\) winds. Without winds the turbulence arises from tidal current interactions with the bottom. With winds there is also a turbulence contribution from wind-induced current shear originating at the surface. Both may be of comparable magnitude. The distribution throughout the estuary is further complicated by geometry leaving simple models like Eq. (5) for conceptual use only.

Sampling salinity and the TKE profiles at a point in the center of the section II channel yields the time evolution of these quantities, given in Figure 7 for the cases of full tides without winds and full tides with 5.0 m s\(^{-1}\) winds. Without winds we see a neap to spring tide modulation such that spring tides are more effective at mixing than neap tides. With winds we see additional modulations. Near the surface a frequency doubling occurs because of the two peaks in wind speed for each wind cycle. Near the bottom maximum TKE occurs when the wind-induced circulation adds constructively with the estuarine circulation. Also, the neap to spring tide modulation is modified by constructive or destructive interference between the tidal and wind-induced currents. Thus, the structure of the turbulent mixing is highly time dependent so that suitable averaging is necessary for estimating the net estuarine circulation and salinity distributions (Weisberg, 1976b).

With the flow field and TKE dependences on rivers, tides, and winds, and the need for a control volume analysis established, we now come to the heart of the matter: how does the system work? In an attempt to answer this question, at least for the CH estuary and in the context of the ECOM3D-si model, we offer a term-by-term analysis of Eq. (9). Detailed results are shown in Figure 8 for the three cases: rivers, plus full tides; rivers, plus full tides, plus 2.5 m s\(^{-1}\) winds; and rivers, plus full tides, plus 5.0 m s\(^{-1}\) winds, and the mean values are summarized for all cases in Table 1. The Figure 8 time series are low pass filtered to

![Figure 5. Net outflow averaged over 80 M\(_2\) tidal cycles and integrated over section II as a function of tidal amplitude and wind speed.](image-url)
remove fluctuations on time scales shorter than 36 hrs. This filters out tides while leaving the wind-induced and neap-spring modulations intact. It also shows that the Table 1 means derive from stationary time series. While each of the terms is evaluated independently some are combined for ease of interpretation. The lead terms are the rate of pressure work (IV), the total derivative of mechanical energy (I + II + III), rate of work by wind stress (V), rate of work by bottom stress (VI), vertical shear production (VII), and the rate of work by buoyancy flux (VIII). Horizontal shear production within the control volume (IX) and divergence by horizontal diffusion across the bounding surfaces (X and XI) are combined together and are small. All of these terms may be gauged against the analysis error (bold black line), defined as the failure of the mechanical energy balance to close.

Consider the simplest case first, that without winds. The pressure work is the driver, and
we see that it is modulated by the neap to spring tide cycle. This is countered primarily by bottom friction. Some of the difference between these two lead terms is accounted for by turbulence production arising from vertical shear and by work against buoyancy arising from the combined effects of vertical advection (Reynolds’ flux) and vertical diffusion. Since the control volume is not the entire estuary there is also a net energy flux divergence. Terms having to do with horizontal mixing (the least justifiable model parameterization) are individually small and they sum to a small time series. The closure error is smaller than either the vertical shear production or the buoyancy flux terms that we will focus on later. Thus, pressure work, material rate of change, and buoyancy flux are on the positive side of the ledger, whereas bottom friction and vertical shear production are on the negative side. The sum of all terms nearly balances to give a stationary solution with an established time-averaged estuarine circulation.

Wind adds complexity, but the basic description is similar. Pressure work continues to arise primarily by tides (although coastal ocean set up adds to this), and an added driver is the oscillatory winds, which most of the time provides a positive work contribution since the near surface flow adjusts rapidly to be in the same direction as the wind stress. However, this added (wind) source of energy increases both the vertical shear production
and the buoyancy flux with offsetting signs. Increasing the wind eventually causes the wind stress work on the sea surface to exceed the pressure work (which also increases). With increasing levels of shear, both the vertical shear production and the buoyancy flux also increase. However, the shear production increases at a larger rate than the buoyancy flux so
Table 1. Summary of terms in the \(te\) equation (9) integrated over the Figure 1b control volume for the cases of river discharges, plus tides; and river discharges, plus tides, plus either 2.5 m s\(^{-1}\), 5.0 m s\(^{-1}\), or 10.0 m s\(^{-1}\) oscillatory (with period of 8 M\(_2\) tidal cycles) winds.

<table>
<thead>
<tr>
<th>Wind speed (m s(^{-1}))</th>
<th>Pressure (IV) (10^5)</th>
<th>Friction (VI) (10^5)</th>
<th>V-shear (VII) (10^4)</th>
<th>Buoyancy (VIII) (10^4)</th>
<th>Wind (V) (10^4)</th>
<th>Total change (I) (10^4)</th>
<th>Diffusion + H-shear (II) (10^4)</th>
<th>Residual (10^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.086</td>
<td>-1.113</td>
<td>-6.112</td>
<td>2.965</td>
<td>0.0</td>
<td>7.066</td>
<td>1.836</td>
<td>5.043</td>
</tr>
<tr>
<td>5.0</td>
<td>2.129</td>
<td>-1.331</td>
<td>-1.578</td>
<td>7.036</td>
<td>1.510</td>
<td>1.076</td>
<td>-1.324</td>
<td>-2.048</td>
</tr>
<tr>
<td>10.0</td>
<td>2.887</td>
<td>-2.005</td>
<td>-5.201</td>
<td>6.099</td>
<td>5.318</td>
<td>8.340</td>
<td>-2.924</td>
<td>-4.831</td>
</tr>
</tbody>
</table>

The terms are mean values averaged over 80 M\(_2\) tidal cycles. The units are kg m\(^2\) s\(^{-3}\), or Watts.
more of the increased work by the wind stress goes into dissipation by turbulence than into changing the buoyancy gradient and hence the driving force of the estuarine circulation. While not shown in the Figure 8 (see Table 1) this becomes most evident for the case of 10 m s$^{-1}$ winds, where the loss by vertical shear production is an order of magnitude larger than gain by buoyancy flux.

Returning to Eq. (5) we can now appreciate how the system works. The conceptual argument is that the time-averaged estuarine circulation is directly proportional to the horizontal buoyancy (salinity) gradient as the driving force and inversely proportional to the vertical mixing coefficient. Both of these quantities depend on the flow field (as forced by rivers, tides, and winds). If the incremental rate of work going into the buoyancy flux exceeds that going into vertical shear production then, all else being equal, the estuarine circulation should increase, or conversely. Figure 9 shows the rate of work by buoyancy flux, the rate of dissipation by vertical shear production, and their ratio for the full tide cases analyzed. By the above argument, as the ratio increases we would expect the estuarine circulation to increase, and conversely. Consistent with Figures 3 and 4 we see a dramatic decrease in the ratio for wind amplitudes exceeding 2.5 m s$^{-1}$. Why the varying tidal amplitude cases have nearly the same transports is less clear, and we must recognize that other terms in the energy balance are also involved. For instance, increasing the tidal amplitude increases the bottom friction as well as the pressure work. Nevertheless, if the underlying argument of estuarine circulation by gravitational convection is correct then the only source for driving the convection is the buoyancy flux. If the rates of dissipation by any process increase faster than the rate of buoyancy flux increase, then the estuarine circulation should decrease. Our analysis argues that the primary process offsetting buoyancy flux is the vertical shear production.

4. Discussion and summary

Beginning with the classical estuarine circulation paradigm of Pritchard (1954) and Hansen and Rattray (1965), that gravitational convection drives a time-averaged two-layered flow with a seaward directed upper layer and a landward directed lower layer, and the estuary classifications of salt wedge to well mixed that follow from this paradigm, we consider the conundrum that at some point of increasing mixing the circulation decreases rather than increases. The underlying assumption for gravitational convection is that the horizontal buoyancy gradient caused by vertical mixing (Reynolds’ flux and diffusion) provides the motive force. Increasing mixing should therefore lead to increasing circulation as the salinity isolines rotate from horizontal under salt wedge conditions to vertical under well mixed conditions. What constrains or even negates the accompanying increase in the buoyancy torque?

Recognizing that estuaries are complex and driven by some combination of river inflows (the buoyancy source) and tides and winds (the turbulence mixing source), we approach this problem through a control volume analysis of the mechanical energy equation. We use the Charlotte Harbor estuary and the ECOM3D-si primitive equation model as the experimental venue and medium, respectively. By forcing the model with rivers only;
rivers, plus tides; and rivers, plus tides and winds, we establish the increase and subsequent decrease in the estuarine circulation and account for this through a term-by-term analysis of the mechanical energy equation. We find that while the rate of work against buoyancy (that sets up the motive force) increases with increasing tide and wind forcing, there comes a point when the relative rate of turbulent energy production exceeds that of the buoyancy flux. Thus, when the ratio of these two terms switches from increasing to decreasing, at a point intermediate between partial to well mixed conditions, the estuarine circulation decreases. This explains why the time-averaged circulation of a well-mixed estuary may be sluggish despite large buoyancy torque.

Figure 9. The averaged (over 80 M_2 tidal cycles) rate of work against buoyancy (upper), rate of dissipation by vertical shear production (middle), and their ratio (lower) as functions of oscillatory wind speed computed over the Figure 1b control volume.
Our results complement those of previous studies on mixing effects for either lock exchange flows or estuaries. The study most closely related to ours is that of Park and Kuo (1996) who also addressed the relative changes in horizontal density gradient to dissipation via a momentum argument. The control volume analysis of the estuarine mechanical energy balance is what distinguishes our work from previous studies. Some other differences also apply. For lock exchange, the initial condition (of different density fluids separated by a removable barrier) is one of maximum buoyancy torque so we would expect any increase in mixing to decrease the exchange flow. For estuaries, the initial condition (a salt wedge with minimal mixing) is one of minimal buoyancy torque so we would expect an initial increase in mixing to increase the circulation as we found here. The eventual preponderance, with increasing mixing, of vertical shear production over buoyancy work is also a consequence of the changing density structure. Under salt wedge conditions static stability impedes shear production. This impediment decreases as the estuary transitions toward well mixed conditions so there comes a point where the shear production increases at a rate much more rapid than the work against buoyancy.

Our results are also complementary with the recent profile dynamics analyses of Geyer et al. (2000), but with a caveat. Their analyses suggest that the Hudson River estuarine circulation can be modeled by an estimate of the bottom drag coefficient, the tidal forcing conditions, and the baroclinic pressure gradient, and that the estuarine circulation is inversely proportional to the amplitude of the tides. They also claim that the estuarine circulation does not depend on mixing of momentum across the pycnocline. The bottom drag, tidal forcing, and baroclinic structure parts are fully compatible with our findings, which can be explained as follows. In steady state, the across-axis component of vorticity derives from a balance between the buoyancy torque integrated over an along-axis plane and the friction forces integrated around the perimeter of that plane. Without winds, this reduces to integrated buoyancy torque opposed by integrated bottom friction so, for a given buoyancy torque, increased tidal amplitude (increased bottom friction) will decrease the estuarine circulation. The caveat is that the set up of the buoyancy torque depends on the full water column response to rivers, tides, and winds. Thus, we are at odds with the second assertion that the circulation does not depend on mixing of momentum across the pycnocline, but this may be estuary dependent. For instance (and with the possible exception of fjords), the deeper the estuary the farther the pycnocline is from the surface and bottom boundary layers and hence the more isolated it is from these sources of mixing. Generalizations from one estuary to another may therefore be tenuous, and there is a need for further parameter studies such as those begun by Hansen and Rattray (1966).

Recent estuary studies are also showing that the circulation is more complicated than the conventional gravitational convection paradigm alone may imply. Rectification of momentum and salt fluxes arising from asymmetries in density field straining and mixing, caused either by boundary layer effects or the constructive and destructive interferences of the various (tide, wind, and gravitational convection) flow components, may result in Reynolds’ transports that may exceed those of the mean gravitational convection. These effects,
resulting in spatially inhomogeneous balances, may necessitate the use of control volume analyses as applied here.

In summary, while we present a plausible explanation for the control on the magnitude of the time-averaged estuarine circulation, it remains to be determined just what defines the state of the estuary. So long as boundary layers are important the frictional parameterizations will be a limiting factor in estuary state variable estimations by models. Our results are model results, not results of nature. Field testing is required, and extension to other estuaries through analyses across Rayleigh number space with varying estuarine geometries (aspect ratios, Froude, and Burger numbers) and forcing functions may be illuminating.

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APPENDIX

Total mechanical energy equations

a. The kinetic energy equation

We define the horizontal kinetic energy \( ke \) as:

\[
ke = \frac{1}{2} \rho_0 (u^2 + v^2)
\]

where \( \rho_0 \) is the reference density; \( u, v \) are horizontal \((x, y)\) velocity components in the Cartesian coordinate system \((x, y, z)\).

With Boussinesq and hydrostatic approximations, the kinetic energy equation is:

\[
\frac{dke}{dt} = -\left( u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_m \frac{\partial ke}{\partial z} \right) + \frac{\partial}{\partial x} \left( A_m \frac{\partial ke}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_m \frac{\partial ke}{\partial y} \right) \\
- \rho_0 K_m \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 - \rho_0 A_m \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \frac{\partial w}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right) \tag{A1}
\]

where \((d/dt) = (\partial/\partial t) + u(\partial/\partial x) + v(\partial/\partial y) + w(\partial/\partial z)\); \( w \) is vertical velocity component; \( P \) is pressure; and \( K_m \) and \( A_m \) are the vertical and horizontal eddy viscosity coefficients, respectively. Eq. (A1) shows that the total rate of change of kinetic energy is controlled by horizontal pressure work, vertical and horizontal diffusion of kinetic energy, and turbulence energy production by vertical and horizontal velocity shears.

b. The potential energy equation

We define the potential energy \( pe \) as:

\[
pe = \rho gz
\]

where \( \rho \) is the perturbation density. The \( pe \) variations follow from the density equation, whence:
\[
\frac{dpe}{dt} = \frac{\partial}{\partial z} \left( K_h \frac{\partial p e}{\partial z} \right) + \frac{\partial}{\partial x} \left( A_h \frac{\partial p e}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_h \frac{\partial p e}{\partial y} \right) + \rho g w - g K_h \frac{\partial p}{\partial z} - g \frac{\partial (K_h \rho)}{\partial z}
\]

(A2)

where \( K_h \) and \( A_h \) are vertical and horizontal diffusivities. Eq. (A2) shows that the total rate of change of potential energy is controlled by vertical and horizontal diffusions of potential energy and buoyancy productions induced by vertical advection and vertical mixing.

c. The total mechanical energy equation

The total mechanical energy \((te)\) is defined as the sum of kinetic energy and potential energy:

\[ te = ke + pe. \]

Adding Eqs. (A1) and (A2) and assuming \( K_h = K_m \), \( A_h = A_m \), we get the total mechanical energy equation:

\[
\frac{dte}{dt} = - \left( u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_m \frac{\partial te}{\partial z} \right) + \frac{\partial}{\partial x} \left( A_m \frac{\partial te}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_m \frac{\partial te}{\partial y} \right) - \rho_o A_m \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right] - \rho_o A_m \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \rho g w - g K_h \frac{\partial p}{\partial z} - g \frac{\partial (K_m \rho)}{\partial z}
\]

(A3)

which is analyzed in Section 2 by integration over the Figure 1b control volume.

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